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INFLUENCE OF MEASUREMENT ERRORS ON UNDERGROUND TOPOGRAPHIC BASES

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Abstract: *Underground topographic bases are of particular importance in the design and management of underground mining works. They can be dependent on the geodetic and topographic system on the surface, but they can also be independent in relation to a particular reference system. In the case of dependent topographic bases, it is preferable, from the point of view of the precisions required for the drawing of mining works in safe conditions, that they are related to two points known by their x, y (fixed) coordinates. The quantities measured in such situations are angles and distances. Measurement errors are transmitted to the determined quantities and consequently it is necessary to analyze the factors with which such a process can be evaluated. The necessary scientific approach is presented below.*
Keywords: *mining surveying, topographic underground networks, errors, topographic measurements*

1. Content of the paper

It is considered the underground topographic base consisting of a polygonal route supported at the ends (fig. 1) [1].

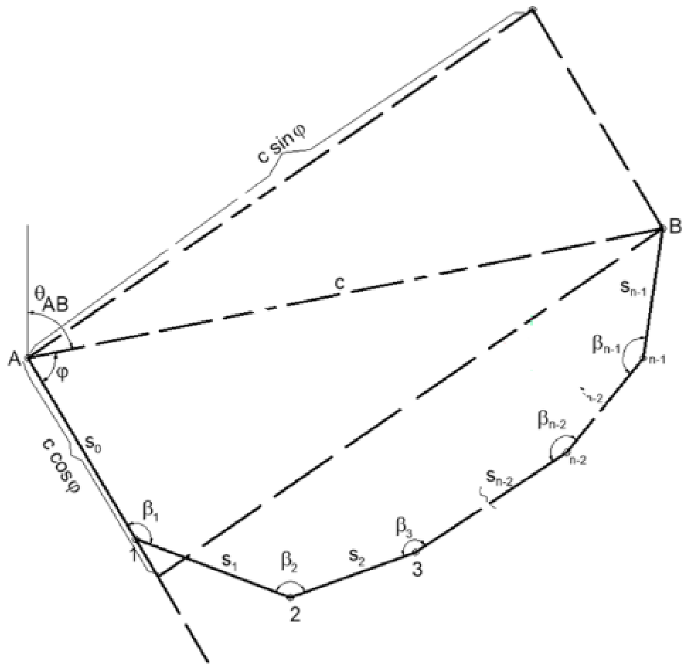

Fig.1. Polygonal route supported at the ends

The topographic base is formed at the level of an open horizon by two vertical wells on which the transmission of coordinates is carried out by the mechanical method [2].

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The resolution of the polygony is carried out using:
The coordinates x_A, y_A, and x_B, y_B of points A and B, as given quantities.

$\beta_1, \beta_2 \dots\dots\dots \beta_{n-1,}\ s_0, s_1 \dots\dots\dots s_{n-1,}$ as measured quantities

The coordinates of the point $B(x_B^{'}, y_B^{'})$ are determined with the equalities:

$$x_B^{'} = x_A + s_0 \cos\theta_1 + s_1 \cos\theta_2 + \cdots s_{n-1} \cos\theta_n \qquad (1)$$
$$y_B^{'} = y_A + s_0 \sin\theta_1 + s_1 \sin\theta_2 + \cdots s_{n-1} \sin\theta_n$$

The orientations of the sides are:

$$\theta_1 = \theta_{AB} + \varphi \qquad \overline{\beta_1} = \beta_1$$
$$\theta_2 = \theta_{AB} + \varphi + \overline{\beta_1} \qquad \overline{\beta_2} = \beta_1 + \beta_2 \qquad (2)$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\theta_n = \theta_{AB} + \varphi + \overline{\beta}_{n-1} \qquad \theta_n = [\beta]$$

The first tie of (1) is developed in the Taylor series and the following are obtained:

$$x_B^{'} = x_B + \frac{\partial x_B^{'}}{\partial x_A} v_{x_A} + \frac{\partial x_B^{'}}{\partial \beta_1} v_{\beta_1} + \frac{\partial x_B^{'}}{\partial \beta_2} v_{\beta_2} + \cdots + \frac{\partial x_B^{'}}{\partial \beta_{n-1}} v_{\beta_{n-1}} + \frac{\partial x_B^{'}}{\partial s_1} v_{s_1} + \qquad (3)$$
$$+ \frac{\partial x_B^{'}}{\partial s_2} v_{s_2} + \cdots + \frac{\partial x_B^{'}}{\partial s_n} v_{s_n}$$

calculated:

$x_B - x_B^{'} = fx$ not closing on x at point B.

but:

$$\frac{\partial x_B^{'}}{\partial x_A} = 0$$

and then:

$$\frac{\partial x_B^{'}}{\partial \beta_1} v_{\beta_1} + \frac{\partial x_B^{'}}{\partial \beta_2} v_{\beta_2} \dots + \frac{\partial x_B^{'}}{\partial \beta_{n-1}} v_{\beta_{n-1}} + \frac{\partial x_B^{'}}{\partial s_1} v_{s_1} + \qquad (4)$$
$$+ \frac{\partial x_B^{'}}{\partial s_2} v_{s_2} + \cdots + \frac{\partial x_B^{'}}{\partial s_n} v_{s_n} + f_x = 0$$

similarly:

$$\frac{\partial y_B^{'}}{\partial \beta_1} v_{\beta_1} + \frac{\partial y_B^{'}}{\partial \beta_2} v_{\beta_2} + \cdots + \frac{\partial y_B^{'}}{\partial s_1} v_{s_1} + \frac{\partial y_B^{'}}{\partial s_2} v_{s_2} + \cdots + f_y = 0 \qquad (5)$$
$$f_y = y_B - y_B^{'}$$

On the other hand, it can be written:

$$\frac{\partial x_B^{'}}{\partial \beta_1} = -s_1 \sin\theta_1 \frac{\partial \theta_1}{\partial \beta_1} - s_2 \sin\theta_2 \frac{\partial \theta_2}{\partial \beta_1} - \cdots - s_n \sin\theta_n \frac{\partial \theta_n}{\partial \beta_1}$$
$$\frac{\partial x_B^{'}}{\partial \beta_2} = -s_1 \sin\theta_1 \frac{\partial \theta_1}{\partial \beta_2} - s_2 \sin\theta_2 \frac{\partial \theta_2}{\partial \beta_2} - \cdots - s_n \sin\theta_n \frac{\partial \theta_n}{\partial \beta_2} \qquad (6)$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\frac{\partial x_B^{'}}{\partial \beta_{n-1}} = -s_1 \sin\theta_1 \frac{\partial \theta_1}{\partial \beta_{n-1}} - s_2 \sin\theta_2 \frac{\partial \theta_2}{\partial \beta_{n-1}} - \cdots - s_n \sin\theta_n \frac{\partial \theta_n}{\partial \beta_{n-1}}$$

But, each orientation also depends on the distances measured, as a result we can write:

$$\frac{\partial x_B^{'}}{\partial s_1} = -s_1 \sin\theta_1 \frac{\partial \theta_1}{\partial s_1} - s_2 \sin\theta_2 \frac{\partial \theta_2}{\partial s_1} - \cdots - s_n \sin\theta_n \frac{\partial \theta_n}{\partial s_1}$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad (7)$$
$$\frac{\partial x_B^{'}}{\partial s_n} = -s_1 \sin\theta_1 \frac{\partial \theta_1}{\partial s_n} - s_2 \sin\theta_2 \frac{\partial \theta_2}{\partial s_n} - \cdots - s_n \sin\theta_n \frac{\partial \theta_n}{\partial s_n}$$

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The orientation of one side is:

$$\theta = \theta_{AB} + \varphi + \bar{\beta}$$

and then:

$$\frac{\partial\theta}{\partial\beta} = \frac{\partial\theta_{AB}}{\partial\beta} + \frac{\partial\varphi}{\partial\beta} + \frac{\partial\bar{\beta}}{\partial\beta} \tag{8}$$
$$\frac{\partial\theta}{\partial s} = \frac{\partial\theta_{AB}}{\partial s} + \frac{\partial\varphi}{\partial s} + \frac{\partial\bar{\beta}}{\partial s}$$

it is noted that:

$$\frac{\partial\theta_{AB}}{\partial\beta} = \frac{\partial\theta_{AB}}{\partial s} = 0 \tag{9}$$

Because the orientation in the general system does not depend on the angles and distances measured, as a result:

$$\frac{\partial\theta}{\partial\beta} = \frac{\partial\varphi}{\partial\beta} + \frac{\partial\bar{\beta}}{\partial\beta}$$
$$\frac{\partial\theta}{\partial s} = \frac{\partial\varphi}{\partial s} + \frac{\partial\bar{\beta}}{\partial s} \tag{10}$$

also:

$$\frac{\partial\beta}{\partial s} = 0$$
$$\frac{\partial\bar{\beta}}{\partial s}=1 \tag{11}$$

as a result:

$$\frac{\partial\theta}{\partial\beta} = 1 + \frac{\partial e}{\partial\beta} \tag{12}$$
$$\frac{\partial\theta}{\partial s} = \frac{\partial e}{\partial s}$$

From figure (1) it can be seen that we can write:

$$tg\varphi = \frac{c\sin\varphi}{c\cos\varphi} = \frac{s_1\sin\bar{\beta}_1 - s_2\sin\bar{\beta}_2 + s_3\sin\bar{\beta}_3 \ldots.}{s_0 - s_1\cos\bar{\beta}_1 + s_2\cos\bar{\beta}_2 - s_3\cos\bar{\beta}_3 + \cdots} = \frac{N}{n} \tag{13}$$

Differentiate the relationship (13) and obtain:

$$\frac{1}{cos^{\,2}\varphi}d\varphi = \frac{Ndn - ndN}{n^2} = \frac{c\sin\varphi dn - \cos\varphi dN}{c^2 cos^2\varphi}$$

or:

$$cd\varphi = \sin\varphi\, dn - \cos\varphi dN \tag{14}$$

We switch from differential to derivative and obtain:

$$c\frac{\partial f}{\partial\beta} = \sin\varphi\frac{\partial n}{\partial\beta} - \cos\varphi\frac{\partial N}{\partial\beta} \tag{15}$$

but:

$$\frac{\partial n}{\partial\beta} = s_1\sin\bar{\beta}_1 - s_2\sin\bar{\beta}_2 + s_3\sin\bar{\beta}_3 \ldots = R\sin\bar{\beta} \tag{16}$$

and:

$$\frac{\partial N}{\partial\beta} = s_1\cos\bar{\beta}_1 - s_2\cos\bar{\beta}_2 + s_3\cos\bar{\beta}_3 \ldots = R\cos\bar{\beta} \tag{17}$$

as such:

$$c\frac{\partial\varphi}{\partial\beta} = -\mathrm{R}\left(\cos\varphi\cos\bar{\beta} - \sin\varphi\sin\bar{\beta}\right) = -R\cos\left(\varphi + \bar{\beta}\right)$$

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and:

$$\frac{\partial\varphi}{\partial\beta} = -\frac{R\cos(\varphi+\bar{\beta})}{c} = -\frac{R'}{c} \quad (18)$$

Substituting in (12) yields:

$$\frac{\partial\theta}{\partial\beta} = 1 - \frac{R'}{c} \quad (19)$$

By applying all derivatives, it results [3]:

$$\frac{\partial\theta_1}{\partial\beta_1} = \frac{\partial\varphi}{\partial\beta_1} = -\frac{R_1'}{c}$$

$$\frac{\partial\theta_2}{\partial\beta_2} = \frac{\partial\varphi}{\partial\beta_1} + \frac{\partial\bar{\beta}_1}{\partial\beta_1} = 1 - \frac{R_1'}{c} = \frac{\partial\theta_3}{\partial\beta_1} = \frac{\partial\theta_3}{\partial\beta_1} = \cdots = \frac{\partial\theta_n}{\partial\beta_1}$$

$$\frac{\partial\theta_1}{\partial\beta_2} = \frac{\partial\varphi}{\partial\beta_2} = -\frac{R_2'}{c} = \frac{\partial\theta_2}{\partial\beta_2} \quad (20)$$

$$\frac{\partial\theta_3}{\partial\beta_2} = \frac{\partial\varphi}{\partial\beta_2} + \frac{\partial\bar{\beta}_2}{\partial\beta_2} = 1 - \frac{R_2'}{c} = \frac{\partial\theta_4}{\partial\beta_2} = \cdots = \frac{\partial\theta_n}{\partial\beta_2}$$

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$$\frac{\partial\theta_n}{\partial\beta_{n-1}} = -\frac{R_{n-1}'}{c} = \frac{\partial\theta_2}{\partial\beta_{n-1}} = \frac{\partial\theta_3}{\partial\beta_{n-1}} = \cdots = \frac{\partial\theta_{n-1}}{\partial\beta_{n-1}}$$

For distances, from (14) by switching to derivatives we have:

$$c\,\frac{\partial\varphi}{\partial s} = \sin\varphi\,\frac{\partial n}{\partial s} - \cos\varphi\,\frac{\partial N}{\partial s} \quad (21)$$

but how:

$$n = s_0 - s_1\cos\bar{\beta}_1 + s_2\cos\bar{\beta}_2 - s_3\cos\bar{\beta}_3 + \cdots$$

it results:

$$\frac{\partial n}{\partial s_0} = 1;\ \frac{\partial n}{\partial s_1} = -\cos\bar{\beta}_1;\ \frac{\partial n}{\partial s_2} = -cos\bar{\beta}_2 \ldots \quad (22)$$

The minus sign is due to the belonging of the orientation to different dials.

Also:

$$N = s_1\sin\bar{\beta}_1 - s_2\sin\bar{\beta}_2 - s_3\sin\bar{\beta}_3 \ldots$$

and:

$$\frac{\partial N}{\partial s_0} = 0;\ \frac{\partial N}{\partial s_1} = \sin\bar{\beta}_1;\ \frac{\partial N}{\partial s_2} = -\sin\bar{\beta}_2 \ldots \quad (23)$$

Replacing with (21) yields:

$$c\,\frac{\partial\varphi}{\partial s_0} = \sin\varphi\cdot 1 - \cos\varphi\cdot 0 = \sin\varphi$$

and:

$$\frac{\partial\varphi}{s_0} = \frac{1}{c}\sin\varphi \quad (24)$$

$$c\,\frac{\partial e}{\partial s} = \sin\varphi\cos\bar{\beta} - \cos\varphi\sin\bar{\beta} = \sin(\varphi - \bar{\beta})$$

$$\frac{\partial\varphi}{\partial s} = \frac{1}{c}\sin(\varphi - \bar{\beta}) = \frac{1}{c}\sin\delta \quad (25)$$

δ – is the angle formed by the sides of the line and side "c"

but:

$$\frac{\partial\theta}{\partial s}=\frac{\partial\varphi}{\partial s}$$

$$\frac{\partial\theta}{\partial s_0}=\frac{\partial\varphi}{\partial s_0}=\frac{1}{c}\sin\varphi \qquad (26)$$

and:

$$\begin{aligned}
&\frac{\partial\theta_1}{\partial s_0}=\frac{1}{c}\sin\varphi=\frac{\partial\theta_2}{\partial s_0}=\frac{\partial\theta_3}{\partial s_0}=\cdots=\frac{\partial\theta_n}{\partial s_0}\\
&\frac{\partial\theta_1}{\partial s_1}=\frac{1}{c}\sin\delta_1=\frac{\partial\theta_2}{\partial s_1}=\frac{\partial\theta_3}{\partial s_1}=\cdots=\frac{\partial\theta_n}{\partial s_1}\\
&\frac{\partial\theta_1}{\partial s_2}=\frac{1}{c}\sin\delta_2=\frac{\partial\theta_2}{\partial s_2}=\frac{\partial\theta_3}{\partial s_2}=\cdots=\frac{\partial\theta_n}{\partial s_2}\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&\frac{\partial\theta_1}{\partial s_{n-1}}=\frac{1}{c}\sin\delta_{n-1}=\frac{\partial\theta_2}{\partial s_{n-1}}=\frac{\partial\theta_3}{\partial s_{n-1}}=\cdots=\frac{\partial\theta_n}{\partial s_{n-1}}
\end{aligned} \qquad (27)$$

The system of error equations has the form:

$$\begin{aligned}
&\left[s_0\sin\theta_1\frac{R_1'}{c}-s_1\sin\theta_2\frac{R_1'}{c}-\cdots-s_{n-1}\sin\theta_n\left(1-\frac{R_1'}{c}\right)\right]v_{\beta_1}+\\
&\left[s_0\sin\theta_1\frac{R_2'}{c}-s_1\sin\theta_2\frac{R_2'}{c}-s_2\sin\theta_3\frac{R_2'}{c}-\cdots-s_{n-1}\sin\theta_n\left(1-\frac{R_2'}{c}\right)\right]v_{\beta_2}+\\
&+\cdots+\left[s_0\frac{\sin\theta_{n-1}}{c}\right]+s_1\sin\theta_2\frac{\sin\theta_{n-1}}{c}+s_2\sin\theta_3\frac{\sin\theta_{n-1}}{c}+\\
&+\cdots+s_{n-1}\sin\theta_n\frac{\sin\theta_{n-1}}{c}\Big]v_{s_{n-1}}+f_x=0
\end{aligned} \qquad (28)$$

$$\left[\frac{y_B-y_A}{c}R_1'-(y_B-y_1)\right]v_{\beta_1}+\left[\frac{y_B-y_A}{c}R_2'-(y_3-y_2)\right]v_{\beta_2}\ ...+\ldots$$

$$...+\left[\frac{y_B-y_A}{c}R_{n-1}'-(y_B-y_{n-1})\right]v_{\beta_{n-1}}+\left[\cos\theta_1\frac{\sin\varphi}{c}+(y_B-y_A)\frac{\sin\delta_1}{c}\right]v_{s_0}+\cdots$$

$$+\left[\cos\theta_2\frac{\sin\varphi}{c}+(y_B-y_A)\frac{\sin\delta_2}{c}\right]v_{s_1}+\cdots+\left[\cos\theta_2\frac{\sin\varphi}{c}+(y_B-y_{n-1})\frac{\sin\delta_{n-1}}{c}\right]v_{s_{n-1}}+f_x=0$$

$$[R_1'\cos\theta_{AB}-(y_B-y_1)]v_{\beta_1}+[R_2'\cos\theta_{AB}-(y_B-y_2)]v_{\beta_2}+\cdots+$$

$$\begin{aligned}
&+[R_{n-1}'\cos\theta_{AB}-(y_B-y_{n-1})]v_{\beta_{n-1}}+\frac{1}{c}[\cos\theta_1\ \sin\varphi+(y_B-y_A)\sin\delta_1]v_{s_0}+\\
&+\frac{1}{c}[\cos\theta_2\ \sin\varphi+(y_B-y_2)\sin\delta_1]v_{s_1}+\cdots+\\
&+\frac{1}{c}[\cos\theta_n\ \sin\varphi+(y_B-y_{n-1})\sin\delta_{n-1}]v_{s_1}+f_x=0
\end{aligned}$$

It is noted:

$$\begin{aligned}
&R_1^y=R_1'\cos\theta_{AB}\\
&\cos\theta_i=\sin\varphi\cos\theta_i
\end{aligned}$$

resulting:

$$\begin{aligned}
&[R_1^y-(y_B-y_1)]v_{\beta_1}+[R_2^y-(y_B-y_2)]v_{\beta_2}+\cdots+\\
&+[R_{n-1}^y-(y_B-y_{n-1})]v_{\beta_{n-1}}+\frac{1}{c}[(\cos\theta_1+(y_B-y_1)]v_{s_0}+\\
&+[\cos'\theta_2+(y_B-y_2)]v_{s_1}+\cdots+[\cos'\theta_n+(y_B-y_{n-1})v_{s_{n-1}}]+f_x=0
\end{aligned}$$

It is similarly applied on the y-axis.

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Conclusions

The paper aims to make a complex analysis of the transmission of the errors of the measured quantities on the quantities determined in the independent simple polygonal routes.

The results of the analysis, which can be obtained analytically, but also graphically, pave the way for determining the accuracies after compensation of the determined quantities. For the tracing of underground works, such analyzes lead to safety in the achievement of objectives of major technical and financial importance.

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